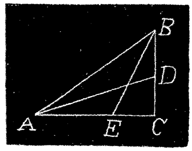
If we take $80p=1248$ or $p=15+3/5$, we find $h=-(4+2/15)$, which gives $a^2=(17/15)^2$. We also have for the three sides, after some easy reductions: 510, 466, 884, and for the medians 659, 683, 208. This is perhaps the simplest case in whole numbers.

43. Proposed by H. C. WILKES, Skull Run, West Virginia.

To find, if possible, a right angled triangle, the bisectors of the acute angles of which, can be expressed by integral whole numbers.

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let ABC be a right triangle, right angled at C, AD the bisector of $\angle A$, and BE the bisector of $\angle B$.



Put $BC=a$, $AC=b$, $AB=c$, $DC=a_1$, $EC=b_1$, $EB=c_1$, and $AD=c_2$. Then $BD=a-a_1$, and $AE=b-b_1$. From geometrical relations we obtain $a^2+b^2=c^2$......(1); $c_1{}^2=ac-b_1(b-b_1)$......(2); $b-b_1 : b_1=c : a$......(3).

From (3) we get $b : b_1=c+a : a$; whence $b_1=ab/(c+a)$, and $b-b_1=bc/(c+a)$.

$\therefore$ $c_1{}^2=ac-ab^2c/(c+a)^2=ac-ac(c^2-a^2)/(c+a)^2=2a^2c/(c+a)$.

By a similar process, we find $c_2{}^2=2b^2c/(c+b)$.

From (1), $c^2-b^2=a^2$, or $(c+b)(c-b)=a^2$. Put $c+b=tp^2$ and $c-b=tq^2$, t, p, and q being any values. Then $a=tpq$, $b=t(p^2-q^2)/2$, and $c=t(p^2+q^2)/2$. Whence $c_1{}^2=2t^2p^2q^2(p^2+q^2)/(p+q)^2$, and $c_2{}^2=t^2(p^2-q^2)^2(p^2+q^2)/4p^2$.

When $p^2+q^2=\square$, $c_2{}^2=\square$, and $c_1{}^2=2\times\square$. When $p^2+q^2=2\times\square$, $c_2{}^2=2\times\square$, and $c_1{}^2=\square$.

$\therefore$ *Both* bisectors cannot be rational; *one* of them will be $\sqrt{2}$ times a number, when the *other* is a rational whole number.

II. Solution by the PROPOSER.

Let bx, by, and $x+y$ be, respectively, the sides and base of a right angled triangle, and let x and y be the greater and less segments of the base cut by the bisector. Then the bisector will be $\sqrt{y^2(b^2+1)}$ and if the bisector be integral, b^2+1 must$=\square$. b must therefore be an improper fraction, and will always be the quotient of the sum of the other two sides divided by the bisected side.

Now let CAB be a triangle, and let $AB=x^2+y^2$, $CA=x^2-y^2$ and $CB=2xy$, $CA+AB/CB=x/y$. $(x/y)^2+1$ may be a square, but $AB+CB/CA=(x+y)/(x-y)$. $[(x+y)/(x-y)]^2+1$ will be a multiple of the $\sqrt{2}$ and cannot be a square.

$\therefore$ If a rational right angled triangle have an integral bisector of one of its acute angles, the bisector of the other acute angle must be a multiple of $\sqrt{2}$ and and cannot be integral.

[Remark.—On page 155, Vol. II. of the MONTHLY, we have, when the sides are 59.4107, 47.4072, 35.8067, the bisectors 40 and 50. It is doubtful whether the sides and bisectors both can be integral. ZERR.]